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Concepts and misconceptions in comprehension of hierarchical graphs ☆

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Abstract

Hierarchical graphs represent relationships between objects (like computer file systems, family trees etc.). Graph nodes represent the objects and interconnecting lines represent the relationships. In two experiments we investigated what concepts are necessary for understanding hierarchical graphs, what misconceptions evolve when some of the concepts are missing and how misconceptions can be prevented through instruction. Participants were taught different amounts of prior knowledge and then had to respond to a multiple-choice questionnaire with interpretive questions about graphs. In Experiment 1, 72 university students received different amounts of instruction about the concepts necessary to interpret hierarchical graphs. Through detailed analysis of readers' wrong responses to interpretive questions we identified a set of misconceptions. Participants maintained fewer misconceptions and performed better if they had been taught more conceptual knowledge. However, their overall performance was poor. In Experiment 2, 85 students were informed about possible misconceptions, in addition to the instruction of conceptual knowledge. With this intervention they obtained an acceptable level of understanding of hierarchical graphs. The discussion of the results draws on theoretical considerations for the evolvement of misconceptions such as

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failure to integrate visual and conceptual information and context specificity of the representation.

Hierarchical graphs represent the relationships between nonnumerical entities or concepts (Butler, 1993). Examples of hierarchical graphs include the structure of computer file systems, preference trees, organisation charts, family trees, and other sorts of conceptual information. Fig. 1 shows an example of a hierarchical graph that represents a hypothetical person's preferences for beverages.

The usefulness of graphs and diagrams for visualisation has often been reported when readers have to learn from texts (e.g., Hegarty & Just, 1993; Mayer & Gallini, 1990; Schnotz & Bannert, 2003; Schnotz, Bannert, & Seufert, 2002). Educators in mathematics have pointed out the importance of teaching students how to interpret diagrams as well as how to produce diagrams from texts (e.g., Barwise & Etchemendy, 1991; Goldin, 1985; Lewis, 1989; Silver, 1982). Novick (2001), Novick and Hmelo (1994), Novick and Hurley (2001), and Novick, Hurley, and Francis (1999) have repeatedly demonstrated the important role of such graphs and diagrams in communication, problem solving and thinking.

Körner (2004) and Körner and Albert (2002b) have recommended design criteria that help optimise hierarchical graph comprehension. In their experiments they relied on response time and eye movement analysis when readers were instructed and trained with the appropriate knowledge to interpret the graphs correctly.

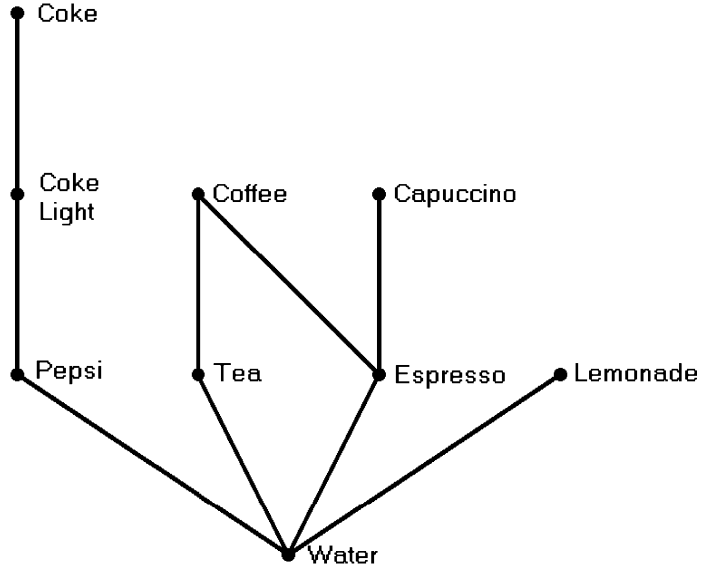

Fig. 1. Example of a hierarchical graph showing preferences between beverages.

What, however, happens when that knowledge is only partially available? The present paper addresses this question and provides a first answer. In general, we assume that in the absence of appropriate knowledge readers may develop wrong conceptions or misconceptions about hierarchical graphs. Misconceptions, also referred to as mind bugs (VanLehn, 1990) or malrules (Sleeman, 1986), are pieces of wrong knowledge; reasoning based on misconceptions leads to consistently wrong problem solutions (see e.g., Preece, 1983; Leinhardt, Zaslavsky, & Stein, 1990, in the context of Cartesian graphs).

Körner and Albert (2002a) provided a first account of the role of misconceptions in comprehension of hierarchical graphs and showed that misconceptions can evolve from insufficient knowledge. In this article, we explore a variety of misconceptions and address the important question of how they can be prevented through appropriate instruction. Therefore, we provided graph readers with different degrees of knowledge for graph interpretation. In Experiment 1 we link wrong graph interpretation to different kinds of misconceptions which evolve due to missing knowledge. In Experiment 2 we directly impart information about misconceptions in order to prevent them.

Misconceptions may arise for a variety of theoretical reasons. In the context of hierarchical graphs the following explanations are pertinent:

(a) Graph comprehension is often regarded as a process that integrates the visual (bottom-up) input from the graph with (top-down) conceptual prior knowledge (e.g., Carpenter & Shah, 1998; Winn, 1994). While a visual representation is being built perceptual properties of the graph like the layout of its elements (their identifiability, proximity, similarity etc.) play an important role. At some point during graph comprehension, abstract higher-level knowledge of the meaning of the graphical elements and the visual representation have to be integrated to achieve comprehension. If visual properties of a graph and prior knowledge are not compatible (or prior knowledge is not available) the integration, and hence comprehension may fail. Moreover, if a graph reader is lacking certain conceptual information he/she may misinterpret visual properties of the graph. Furthermore, such properties may even give rise to inappropriate concepts or misconceptions. As will be shown and discussed later, hierarchical graphs are a good example for the interplay between bottom-up perceptual processes and conceptual knowledge.

(b) Another explanation for graph miscomprehension may be the misapplication or erroneous transfer of conceptual knowledge of other representations. Novick and Hmelo (1994) have shown that problem-solvers can transfer representations between problems. Analogously, they may apply rules from a different graphical representation to hierarchical graphs. Note that hierarchical graphs are visually very similar to representations such as search trees, semantic networks or concept maps; yet, the represented conceptual information can differ considerably. The correct interpretation of hierarchical graphs requires correct identification of the type of graph and selection and application of the appropriate set of concepts or rules of interpretation. In the absence of relevant interpretation knowledge, the (erroneous) transfer of known interpretation rules from visually similar representations might be tempting for a graph reader and, in consequence, lead to misconceptions.

(c) Finally, hierarchical graphs can represent information from a variety of different domains; recall the introductory examples: computer file systems, family trees, organisational structures or preference trees.
Graphs visualise only certain aspects of a domain which is usually richer and may contain additional relations between objects. Consider the object coffee which is preferred to both tea and espresso in Fig. 1. It is tempting to (wrongly) regard tea and espresso as 'equal' in some sense; all the more since they are drawn at the same horizontal level. Now replace coffee with vice-chancellor and tea and espresso with assistant and dean of faculty, respectively (as in an university organisation chart). In this context, the misconception that the respective objects (the assistant and the dean) are 'equal' would rarely arise. The example shows that labels of a graph node can convey additional information that depends specifically on the context. A number of researchers have demonstrated the context dependency of graphing and graph reading in investigations using experts as well as adults with different academic backgrounds (e.g., Roth & Bowen, 2001; Stern, Aprea, & Ebner, 2003).

In this paper, we concentrate on the identification of misconceptions based on graph readers' performance and we aim to prevent them. In the discussion we will return to the above considerations for the interpretation of the main results.

Since conceptual knowledge is crucial for graph comprehension, we have to specify what concepts a graph reader needs in order to interpret a hierarchical graph correctly (see Table 1). In a hierarchical graph (see Fig. 1), a labelled node represents an object (beverage) and a line connects two nodes if the represented objects are in the specified hierarchical (preference) relation. A line is drawn only between neighbouring nodes, and omitted for all other nodes. The graph is directed, that is, if an object is superordinate to another object its node is drawn vertically above that object's node (concept C1). Note that not all possible pairs of beverages are comparable with respect to the preference relation. For example, Coke is preferred to Coke light but not to coffee. Such pairs of objects are called incomparable (C2). Comprehension of hierarchical graphs rests on the concepts of directedness and

Table 1
Concepts needed for comprehension of hierarchical graphs

Concept	Explanation
C1	Directedness: the node of a superordinate object is drawn above the node of a subordinate object.
C2	Comparability: in order to compare two objects their nodes have to be connected by a line.
C3	Domination: an object dominates all the other ones if its node is drawn above all other nodes and is connected to all the other nodes.
C4	Maximality: the node of a maximal object is not connected to any other node drawn above of it.

comparability. Furthermore, there are certain objects with a designated meaning. An object is said to dominate other objects if its node is drawn above all the other nodes and if it is connected to all the other nodes (C3). In Fig. 1, there is no such object; water, however, is the opposite of a dominating object, i.e. it is subordinate to all the other objects. Finally, an object is called a maximal object if its node is not connected to any other node drawn above of it (C4), i.e. if there is no object which is more preferred. Coke, coffee, cappuccino and lemonade are all examples of maximal objects in Fig. 1. For the proper comprehension of the information contained in hierarchical graphs, a reader needs to know these four concepts.

Note that concepts C3 and C4 imply knowledge of C1 and C2; they specify the properties of dominant and maximal objects but their definition rests on the concepts of directedness and comparability.

What will happen if the appropriate knowledge is not available? We assume that misconceptions evolve which can arise for a variety of reasons as discussed above. Table 2 lists seven misconceptions that we investigated in the present experiments. The list is based on prior research (Körner & Albert, 2002a) and by no means exhaustive. The *height* misconception (M1) occurs if a participant assumes a node drawn above another one is superior regardless of the nodes being interconnected by a line or not. Assuming that cappuccino is preferred to lemonade is an example of such a misconception. If two nodes which are drawn at the same height in the graph (e.g., Pepsi and tea in Fig. 1) are mistaken as equal or equally preferred we assume that a reader has the *equality* misconception (M2). The misconception of *false domination* (M3) is present if a single node which is drawn above all the other graph nodes is mistaken as a dominant object, e.g., Coke in Fig. 2. If two nodes with the same upper and lower neighbours (like tea and espresso both of whose neighbours are coffee and water) are mistaken as equivalent the misconception of *equality through neighbours* (M4) is present. Note that for this misconception to occur the 'equivalent' nodes need not be drawn at the same height as in Fig. 1. If a node is mistaken as superior to another one because there are more nodes drawn below of it

Table 2
Misconceptions investigated in the experiments

Misconception	Explanation
M1	Height: an object whose node is drawn above another object's node is superior to that object.
M2	Equality: two objects whose nodes are drawn at the same height are equal.
M3	False Domination: if a single node is drawn above all the other nodes the represented object is superior to all the other objects.
M4	Equality through neighbours: two objects whose nodes share the same upper and lower neighbours are equal.
M5	Number of subordinate nodes: an object whose node has more nodes drawn below of it than another node is superior to the object represented by that node.
M6	False maximality: an object whose node has no other (unconnected) nodes drawn above of it is superior to objects whose nodes have other nodes drawn above of them.
M7	Equality of maximal nodes: if there are several maximal objects, they are regarded as equal.

the misconception of the *number of subordinate nodes* (M5) is present. If a node (like coffee) is regarded as being preferred to another one (like coke light) because there is no node drawn above coffee but there is one above Coke light (Coke), then the misconception of *false maximality* (M6) is present. Finally, if a reader assumes all maximal nodes as being equal (like coke, coffee, cappuccino, lemonade), the misconception of *equality of maximal nodes* (M7) is present.

Note that, in principle, all of the misconceptions emanate on the basis of lacking or ignored comparability knowledge (C2). If a reader strictly observes the concept that two nodes have to be connected by a line in order to be comparable, then he/she can infer that any superior or equality statements between unconnected nodes as implied by the misconceptions are impermissible. Indeed, Körner and Albert (2002a) found that readers who were taught the concepts C1 and C2 made substantially more correct responses to interpretive questions about graphs than those readers who knew only of C1. Even with comparability knowledge taught, however, the participants provided a substantial amount of incorrect responses. Without this kind of knowledge graph comprehension might not be possible.

1. Experiment 1

In this experiment we instruct knowledge of directedness and comparability (C1 and C2) to all of our participants in the experimental groups but vary the amount of knowledge that is instructed in addition (concepts C3 and C4). In general, we expect that the number of correct responses increases with the amount of instructed knowledge. Since knowledge of domination (C3) and maximality (C4) relies on directedness and comparability instruction of those additional concepts should reinforce and improve the knowledge of the basic concepts C1 and C2.

Conversely, we expect the number of misconceptions to increase as the amount of instructed knowledge decreases. Specifically, if the concept of domination is not instructed we expect the misconception of false domination to be more frequent.

1.1. Method

1.1.1. Design

The amount of instructed knowledge was varied between participants on three levels (knowledge groups). We instructed the basic concepts of directedness (C1) and comparability (C2) to group C1C2. Group C1C3 was taught the same two concepts and, in addition, the concept of domination (C3); group C1C4 was taught the previous three concepts and the concept of maximality (C4), as well. There was also a control group which was instructed no relevant knowledge.

Participants' responses to a multiple-choice questionnaire with interpretive questions about hierarchical graphs were collected.

1.1.2. Participants

There were 72 psychology students, that is, 18 per group (63 female, 9 male) with an average age of 22.7 (range 18–39); they participated for class credit.

1.1.3. Materials

We constructed 40 multiple-choice problems which consisted of a graph, an interpretive question and five statements as possible answers to the question. We used four different graphs combined with different questions. We chose hypothetical preferences for vacation cities as domain. Therefore, the graph nodes were labelled with names of European capital cities. An interpretive question asked either for the relation between objects (“What is the relation between Rome and Madrid?”) or for a property of a certain object (“Is London the most preferred city?”). A question was followed by a list of five statements one of which was the correct answer. The statements included a short explanation which allowed us to link a respective wrong statement to one (and only one) misconception. For example, the statement “Rome is preferred to Madrid because there are more cities drawn below Rome than there are cities drawn below Madrid” was indicative for misconception M5 (number of subordinate nodes). A statement like “London is the most preferred city because it is drawn above all the other cities” was indicative for misconception M3 (false domination) given that London was not connected to all of the other nodes in the graph. To keep the guessing probability constant across all problems (0.2) the list of possible answers could include wrong statements that were not indicative of a misconception (unspecified wrong statements).

Across all problems the number of statements, which were indicative of a certain misconception varied between 6 and 18 statements. Originally, we investigated 14 different misconceptions but report data only for the seven most frequent misconceptions which were also included in Experiment 2 (see Table 1). The problems were filed together in a small leaflet in random order.

1.1.4. Procedure

Participants were tested in groups of 1 to maximally 4. In the beginning of the experiment, participants read written instructions in which it was stated that, in general, preferences for vacation cities can be visualised by means of hierarchical graphs. The following information differed between the knowledge groups (C1C2, C1C3, C1C4). We explained the respective concepts C1 to C4 in detail and illustrated them with two example graphs that were not used in the questionnaire later on. The control group did not receive such information. After having read the instructions participants started working on the problems.

1.2. Results and discussion

For each participant we determined the percentage of correct responses and averaged across participants of a group (see Table 3).

A one-way ANOVA showed that the percentage of correct responses varied between the four groups ($F(3,68) = 14.2$, $p < 0.01$); Scheffé tests revealed that each of the three knowledge groups performed significantly better than the control group ($p < 0.05$). Control group performance was not significantly different from chance (i.e. 20%): $t(1,17) = 2.1$, $p > 0.05$, suggesting that these participants had no relevant prior knowledge. The performance of Group C1C2, however, was clearly above

Table 3
Mean percentage correct (SD) for groups in Experiment 1

Group	Mean (SD)
Control	13.1 (13.9)
C1C2	36.0 (23.2)
C1C3	48.6 (23.5)
C1C4	56.0 (22.7)

chance ($t(1,17) = 2.9$, $p < 0.01$). These results showed that the different instructions had the expected effect on participants' performance and therefore provided the basis on which misconceptions could be investigated.

The number of statements which indicated a certain misconception differed in the questionnaire (see above); therefore, we based the following analysis on percentages of responses given to all statements which were indicative for a certain misconception. For example, if there were 10 statements indicating a misconception and a participant chose one of those statements eight times this corresponded to 80%. Fig. 2 shows the misconception distribution averaged across individual percentages for each of the groups. A 4 × 7 ANOVA for groups and misconceptions as within-subjects factor showed that the presence of misconceptions differed substantially between groups ($F(3,68) = 10.8$, $p < 0.01$); for example, the control group maintained 26.4% misconceptions on average compared with 8.2% of group C1C3. The misconceptions themselves were very unevenly distributed ($F(6,408) = 25.9$, $p < 0.01$). The equality misconception (M3) was present in 28.3% of all cases while the misconception of the number of subordinate nodes (M5) was present only in 4.1% of all cases. Therefore, the interaction was also significant ($F(18,408) = 6.5$, $p < 0.01$) suggesting that the distribution of misconceptions varied depending on both the type of misconception and group (see Fig. 2).

When we restricted the same kind of analysis to the three most frequent misconceptions (M1, M2, M3) and the three knowledge groups we observed a clear

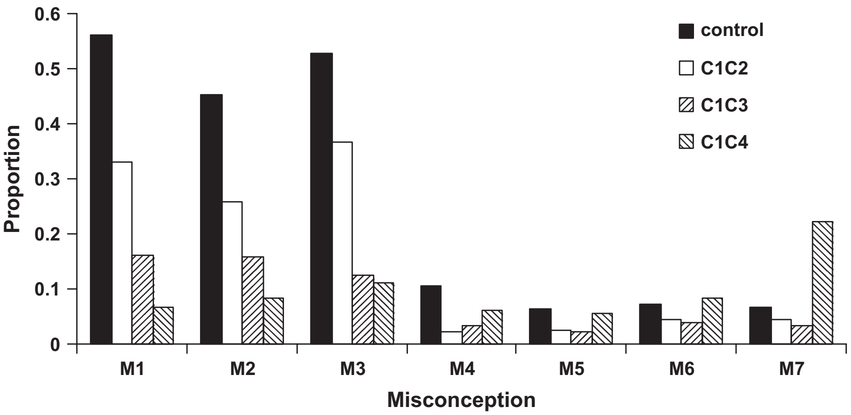

Fig. 2. Proportion of misconceptions for different groups in Experiment 1.

difference between groups ($F(2,51) = 5.1$, $p < 0.01$) and no other effects. To ensure that the proportion of misconceptions decreased as more knowledge was made available we regressed the groups on the misconception proportion for the three misconceptions ($R^2 = 0.14$, $\beta = -0.38$, $p < 0.01$ for M1; $R^2 = 0.12$, $\beta = -0.35$, $p < 0.05$ for M2; $R^2 = 0.12$, $\beta = -0.35$, $p < 0.01$ for M3). To test the prediction that missing knowledge of the concept of domination (C3) provokes the misconception of false domination (M3) we carried out a one-way ANOVA for M3. As expected, the groups differed with respect to the presence of that misconception ($F(2,51) = 4.7$, $p < 0.05$); Scheffé tests showed that this misconception was less pronounced for group C1C3 (which was taught the domination concept) compared with group C1C2 ($p < 0.05$) while there was no difference between group C1C3 and group C1C4 ($p > 0.05$). This result indicates that domination knowledge can reduce the presence of the related misconception of false domination.

With respect to misconceptions M4, M5 and M6, a detailed analysis was not feasible because there was a clear floor effect for these misconceptions; they were present in 10% or less of all cases. Concerning misconception M7 we observed an unexpected result: participants of group C1C4 who had been taught more concepts than any of the other groups maintained this misconception more often (22.2%) as shown by a one-way ANOVA ($F(2,51) = 5.0$, $p < 0.05$) and post hoc tests contrasting that group with the other knowledge groups ($p < 0.05$). We assume that a misinterpretation of the instructed knowledge about maximality (C4) is responsible for this result. In the instruction we had illustrated the property of maximal objects (i.e. their nodes are not connected to other nodes drawn above) using two such nodes in the same graph. This was done to emphasise the possibility of more than one maximal object. By doing so we may have led our participants to believe that if several nodes share this property they can be regarded as equal (misconception M7).

Experiment 1 supports the following conclusions: (1) the presence of the misconceptions of height, equality and false domination depends strongly on the amount of instructed knowledge. The more knowledge was taught the less pronounced were these misconceptions. (2) Misconceptions M4, M5, and M6 are comparatively infrequent. (3) Despite having been taught about maximal nodes participants maintain a misconception about these nodes, probably due to a misinterpretation of their definition in the instruction. (4) Having no knowledge of hierarchical graphs has devastating consequences if readers need to interpret them: participants of the control group performed on chance level. This result emphasises that hierarchical graphs are by no means self-explanatory. Even with a fair amount of instruction of all the concepts necessary for proper graph comprehension (group C1C4) performance is hardly better than 50% (see Table 3). Therefore, in Experiment 2, we also imparted knowledge about misconceptions.

2. Experiment 2

In this experiment we aim to replicate the main results from Experiment 1, i.e. to confirm misconceptions M1, M2, and M3 as the most prominent ones and to show that there is hardly any graph comprehension without instruction (control group). The main purpose of this experiment is to prevent misconceptions through appropriate instruction and thus establish an adequate level of understanding.

2.1. Method

2.1.1. Design

The experiment was based on a 2 × 2 between-subjects design. The amount of instructed knowledge varied on two levels: either the concepts C1, C2, and C3 were taught (groups C1C3 and M/C1C3) or all the concepts C1 to C4 were taught (groups C1C4 and M/C1C4). Information about misconceptions also varied on two levels: two groups with different knowledge were informed about misconceptions (groups M/C1C3 and M/C1C4) while the other two groups were not (group C1C3 and group C1C4). There was also a control group which was taught no relevant knowledge of concepts or of misconceptions.

Participants' responses to a multiple-choice questionnaire with interpretive questions about hierarchical graphs were collected.

2.1.2. Participants

There were 85 first-year psychology students (17 in each of the five groups) participating in the experiment for class credit. Sixty-nine participants were female and 16 male with an average age of 21.9 (range 18–35). They had not participated in Experiment 1.

2.1.3. Materials and procedure

We constructed 20 problems (graph-question combinations) using the graphs from Experiment 1. There were four statements to choose from as possible answers to the question. The number of statements which were indicative for a certain misconception varied between 5 and 7.

Again, a question asked either for the relation between nodes of the graph or for a property of a certain node in the context of vacation preferences. In the instruction we explained the respective concepts and misconceptions in detail to the four experimental groups and illustrated them with examples. The control group did not receive such information. The rest of the procedure was identical to Experiment 1.

2.2. Results and discussion

For each participant we determined the percentage of correct responses and averaged across participants of a group (see Table 4). A 2 × 2 ANOVA for the factors instruction and misconception information showed that warning participants about possible misconceptions substantially improved their performance ($F(1,64) = 22.0$, $p < 0.01$). We found no other effects. The control group performance did not differ from 25% chance performance ($t(1,16) = 1.0$, $p > 0.05$).

Fig. 3 shows the distribution of misconceptions across the four experimental groups. Again misconceptions M1, M2, and M3 were the most prominent ones.

Table 4
Mean percentage correct (SD) for groups in Experiment 2

Group	Mean (SD)
Control	19.7 (21.5)
C1C3	53.2 (25.6)
C1C4	53.8 (19.2)
M/C1C3	72.4 (25.1)
M/C1C4	84.4 (16.2)

A $2 \times 2 \times 7$ ANOVA for instruction, misconception information and the misconceptions as within-subjects factor yielded a clear effect of the misconception information ($F(1,64) = 10.9$, $p < 0.01$) and an interaction of misconception information and the misconceptions showing that informing participants of misconceptions affected their presence differently ($F(6,384) = 2.5$, $p < 0.05$). Analysing this interaction with a simple main effects analysis for the misconceptions indicated that misconception information reliably reduced misconceptions M1 ($F(1,64) = 6.1$, $p < 0.05$) and M2 ($F(1,64) = 18.7$, $p < 0.01$).

A $2 \times 2 \times 3$ ANOVA for instruction, misconception information and the three most frequent misconceptions M1, M2, and M3 yielded a clear effect for the misconception information ($F(1,64) = 12.7$, $p < 0.01$) showing that it reduced the average percentage of these misconceptions from 18.4% to 6.7%. We found no other effects.

From Fig. 3 it appears as if we had replicated the unexpected result from Experiment 1 regarding the equality of maximal nodes (M7) for group C1C4; however, in a 2×2 ANOVA the factor misconception information slightly failed to

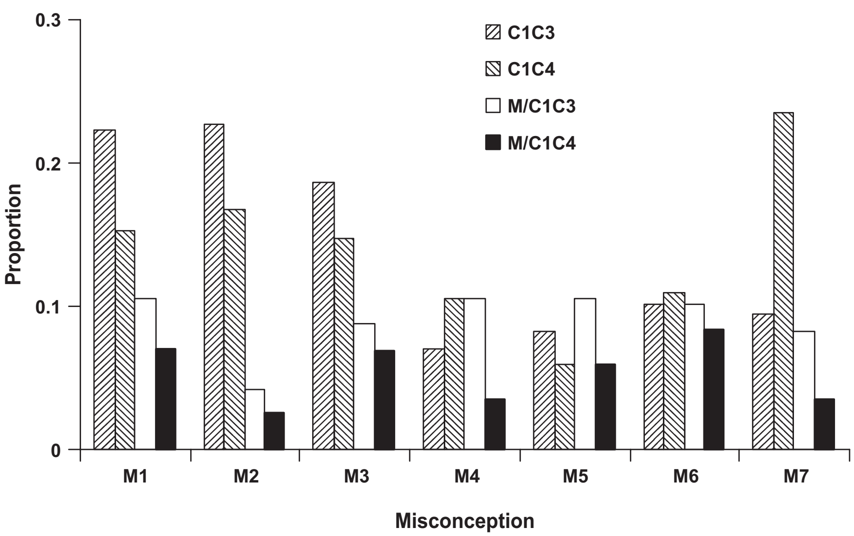

Fig. 3. Proportion of misconceptions for different groups in Experiment 2.

reach significance ($F(1,64) = 3.7$, $p = 0.6$), although the proportion of that misconception dropped from 23.5% to 3.5% when we had warned participants with knowledge of concepts C1 to C4. Thus, there is only limited evidence that misconception information remedied the possible ambiguity in the instruction about concept C4.

Experiment 2 has provided three major results: (1) the misconceptions of height, equality and false domination are the most common ones. (2) They can be effectively prevented by informing participants about misconceptions in addition to the mere instruction of the concepts relevant for graph comprehension. (3) Not knowing the concepts and being uninformed of misconceptions leads to graph comprehension on chance level.

3. General discussion

In two experiments we investigated what concepts are necessary for understanding hierarchical graphs, what misconceptions evolve when some of the concepts are missing and how these misconceptions can be prevented through adequate instruction. The main results are: (1) the more knowledge that is available to graph readers the better their graph comprehension is. (2) The more knowledge they have the less likely they are to maintain misconceptions. (3) The misconceptions of height, equality and false domination are the most prominent ones. (4) In order to obtain an acceptable level of understanding of graphs it is essential that graph readers are informed about possible misconceptions in addition to mere instruction of the concepts of proper graph reading. (5) Without knowledge of either the concepts or possible misconceptions in graph comprehension there is no understanding of hierarchical graphs.

The prevalence of the misconceptions of height, equality and false domination can be interpreted with regard to the considerations set forth in the introduction.

A central theoretical assumption of graph research is that graph comprehension is both a bottom-up as well as a top-down process. Bottom-up processing is driven by perceptual features of the graph while top-down processing relies on (prior) knowledge. If perceptual features are compatible with the message of the graph, this will facilitate integration of information, and hence comprehension. Incompatibilities will impair comprehension or may even lead to misconceptions as shown in our experiments. This is also known as the compatibility principle (e.g., Kosslyn, 1989, 1994) that states that the layout of graphical elements should reflect the meaning of the graph.

The foremost conceptual property of hierarchical graphs is that they represent an asymmetrical relationship between objects (directedness C1). The fact that nodes of superordinate objects are drawn above those of subordinate ones visualises this property. In the sense of the above principle this is a compatible representation because it exploits the culturally prevalent convention that something that is located above something else is "superior" or "better". Being drawn above in a hierarchical graph is, however, only a necessary condition that a node is superordinate and as such it is not sufficient; the node must also be connected to another node by a line. This is the graphical representation of the concept of comparability (C2). Graph readers without comparability knowledge probably mistake the vertical arrangement of nodes as a sufficient criterion for superiority as stated by the height misconception. When participants in the control groups (who were given no instructions at all) were asked "Is city A preferred to city B?" (with A and B not being connected in the graph) they typically responded 'yes' if city A was drawn higher up in the graph. Since control participants were not taught the concept of directedness, this finding emphasises the power of the graphical representation of this concept. Without having been taught these participants assumed that higher-up nodes represent superordinate objects. In our experiments we found the height misconception for participants with comparability knowledge as well. This may indicate that the visualisation of comparability is not as straightforward as is the visualisation of directedness.

Another prominent misconception in the present experiments was the misconception of equality: two (or more) objects whose nodes are drawn at the same height in the graph are regarded as equal (and thus, comparable). It occurs when participants have no knowledge of comparability or when this knowledge is not being applied. It illustrates again that missing conceptual knowledge may be replaced by misconceptions that arise from properties of the perceptual representation. By convention, graph nodes are drawn at the same height or the same level when they represent neighbours of the same superordinate or subordinate object (see Fig. 1). Specialists from mathematics and computer science regard this property of levels as very important because it makes the property of being super-/subordinate to the same object explicit (see Pelc & Rival, 1991). Usually, the overall structure and appearance of graphs profits from the levels property; levelled graphs look less cluttered. Therefore, one might assume that a levelled graph facilitates comprehension. In our experiments, however, readers with limited knowledge found the visual property of levels so compelling that they (over-) interpreted it as a new relation existing between objects with levelled nodes, namely that they are equal. This is consistent with recent research (Lowe, 2003) that shows that inexperienced readers of dynamic graphics typically process those graphics in a bottom-up (perceptually driven) manner which can lead to miscomprehension.

The third frequent misconception was false domination: If a single node is drawn above all the other nodes the respective object is superior to the other objects. This misconception can be regarded as a special case of the height misconception. Again, the height of a node is mistaken as a sufficient criterion for the represented object to be superordinate when comparability knowledge is not available or neglected. The perceptual feature of a single node being the top-most graphical element might be visually so salient that, during later processing, it is mistaken as a sufficient criterion for a dominant object.

Preference relations are only one kind of information that can be visualised in a hierarchical graph. Some of the misconceptions investigated in the present experiments may be specific to the preference context and may not occur in a different context. For example, we can represent who is superior to whom in a company with the help of a hierarchical graph (organisation chart). In such a context, it appears unlikely that a reader would conclude that employees drawn at the same horizontal level are ‘equal’ (cf. the university example in the introduction). The names of the objects (employees) bear contextual meaning that helps to understand their relations and to preventing misconceptions. The organisation context helps to understand the incomparability property inherent in hierarchical graphs. This property is much less obvious in a preference context. Roth and Bowen (2001) have emphasised that graph interpretation skills are highly dependent on the context and on the familiarity of the domain to which the graph refers. Yet, transfer of representations across domains is possible if the reader succeeds in abstracting from the specific context (Stern et al., 2003). Therefore, hierarchical graphs might be well suited to study the combined effects of domain specificity and transfer across domains. For example, graph readers could be trained with graphs in a domain that likely prevents certain misconceptions; later on the same readers have to interpret graphs in a different domain that usually provokes such misconceptions, or vice versa.

From our experiments we cannot rule out that participants misinterpreted hierarchical graphs because they misapplied knowledge of other representations like concept maps or search trees. A more refined questionnaire and better control of participants’ pre-experimental knowledge might shed some light on this issue in the future.

On first sight, hierarchical graphs look easy enough to understand (see Fig. 1). However, in the present experiments we found that participants in control groups, i.e. graph readers who had not received any instructions performed on chance level. This result emphasises the well-known claim of visual literacy that the understanding of any pictorial representation has to be taught and learned in the same way as the understanding of language is being taught. A good language teacher does not only teach the concepts and rules (grammar etc.) of language but also points out possible pitfalls and common mistakes. Similarly, the information about possible misconceptions in addition to the instruction about the concepts in graph reading paid off for the participants of our experiments.

Finally, what practical conclusions can be drawn from the present experiments? (1) Instruction is crucial. Hierarchical graphs are by no means self-explanatory. Without instruction there is only very limited understanding of hierarchical graphs. The instruction of the necessary concepts for proper graph comprehension has to be done carefully. In Experiment 1, the misconception of maximal objects was probably induced by improper instruction of the respective concept. (2) Mere instruction of concepts is not good enough. This was shown by the poor performance of participants with well-taught concepts in both experiments. Only when information about misconceptions was provided in addition to conceptual knowledge graph comprehension approached an acceptable performance level. The misconceptions of height, equality and false domination are most prominent and have to be prevented with precedence. (3) It may be worth reconsidering the drawing of nodes at the same height. This comes with the cost of cluttered graphs. However, at least in the context of preference, it could help preventing the equality misconception. Of course, the trade-off between misconception prevention and more regular looking graphs is, amongst other topics, subject to further research.

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